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Kinetic Effects in High Gain Free-Electron Lasers

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Abstract. A formalism based on the Vlasov-Maxwell system has been developed that provides a fully kinetic description of a free-electron laser (FEL) operating in the high gain regime. The analysis allows for guiding of the optical beam through the gain process and guiding of the electron beam by the weak focusing provided by realistic wiggler gradients. Thus, betatron oscillations and emittance are naturally included. Additionally, intrinsic energy spread as well as energy spread due to space charge effects is included. The analysis predicts a novel electron beam equilibrium flow in which the effect of wiggler gradients tends to cancel that due to space charge, leading to a flow with reduced axial velocity spread. Since the FEL mechanism is sensitive to the axial velocity spread on the beam, this special flow appears to be useful in enhancing FEL gain. This and other issues related to FEL operation are discussed in this paper. It is shown that the scaled thermal velocity $S = v_{th} / (v_b - v_{ph})$ is a useful measure of beam quality in the context of FELs. Here, v_{th} is the thermal velocity on the beam, v_b is the beam velocity and v_{ph} is the phase velocity of the ponderomotive wave. The scaled thermal velocity depends on beam emittance, energy spread and electron density and is a useful indicator of beam quality since it provides a measure of the thermal spread as observed in the phase velocity frame. Analogies between the high gain FEL and plasma two-stream instability are discussed.

INTRODUCTION

After the first experiments in the late seventies demonstrating lasing in the infrared the free-electron laser (FEL) became the alternate concept to all coherent radiation sources from microwaves to X-rays [1]. This was motivated by the FEL's tunability, wavelength accessibility and potential for high power operation. After 20 years, the focus of FEL research for scientific applications has shifted to UV and X-ray wavelengths where coherent sources are rare or nonexistent. The accelerator and beam quality requirements are similar to a linear collider. The lack of mirrors in the hard X-ray regime has focused the effort to single pass high gain FEL operation. Figure 1 is a sketch of such an FEL. The lower panel in Fig. 1 shows the exponential gain and nonlinear saturation of this collective instability by trapping of the beam

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particles in the potential wells of the space charge waves. Similar exponential growth and nonlinear trapping of the beam particles has been observed in a two-stream instability [2]. The form of the dispersion relation for the high gain FEL and the two-stream instability is similar. The two-stream instability in the cold beam limit is a hydrodynamic, or fluid, instability. The exponential growth is due to coupling of positive and negative energy modes on the beam by the plasma. In the kinetic, or warm

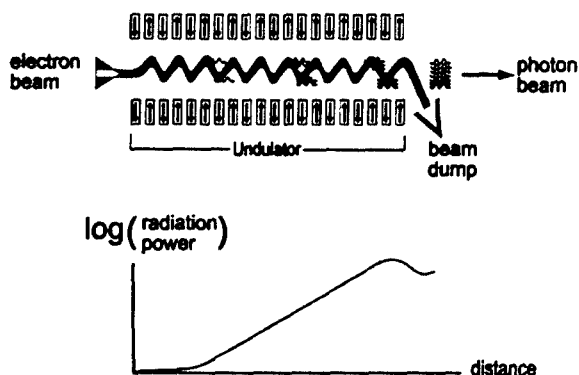


FIGURE 1. Upper panel: FEL amplifier configuration. Lower panel: Growth of radiation in SASE regime. (tesla.desy.de)

beam limit, the real part of the plasma dispersion relation remains unchanged by the beam, and the growth is by inverse Landau damping. O'Neil and Malmberg [3] have examined this change in topology of the dispersion from the fluid to the kinetic limit. They found that when the scaled thermal velocity $S = (v_{th} / v_b)(2n_0 / n_b)^{1/3}$ is small compared to unity the instability is in the fluid limit. When S is approximately equal to or greater than unity, the instability is in the kinetic limit. Here, v_{th} is the thermal velocity on the beam, v_b is the beam velocity, n_0 is the plasma and n_b is the beam density. More generally this can be written as $S \equiv v_{th} / (v_b - v_{ph})$, which we adopt as the FEL beam quality [4]. The specifics of FEL physics are contained in v_{ph} , the phase velocity of the ponderomotive wave. For the two-stream instability these two definitions of S are equivalent. In addition to the usual contributions to the effective thermal velocity from the beam emittance and energy spread, this includes the FEL physics through v_{ph} . Conceptually S may be viewed as a measure of how the thermal spread on the beam as viewed from the ponderomotive/space charge wave frame of reference.

KINETIC ANALYSIS

The electron beam couples with the beat wave arising from the radiation field and the wiggler field. The beat wave is also referred to as the ponderomotive wave.

The radiation field can be due to either small amplitude injected beam ('seed') radiation or spontaneous emission. The absence of a source at X-ray wavelengths, for example, means that FEL amplifiers in this regime run in the self amplified spontaneous emission (SASE) mode; Fig. 1. In SASE the growth of the radiation at first is essentially linear (the 'lethargy' regime) as a rough phase relationship is established with the electrons. Then the growth becomes exponential and is accompanied by strong modulation of the electron beam on the optical scale. Growth in the exponential regime leads to extraction of energy from the electron beam, which is slowed down and eventually trapped in the ponderomotive buckets. In complete analogy with two-stream instability, in the saturated regime trapped particle oscillations lead to characteristic oscillations in the optical field intensity.

The resonance condition in the FEL interaction is expressible as

$$\omega - (k + k_w)v_z = 0, \tag{1}$$

where v_z is the axial electron velocity, ω and k are the frequency and wavenumber of the radiation field, $k_w = 2\pi/\lambda_w$ is the wiggler wavenumber and λ_w is the wiggler period. It follows from Eq. (1) that the beat (ponderomotive) wave phase velocity is

$$\beta_{ph} \equiv \frac{v_{ph}}{c} = \frac{\omega}{k + k_w}. \tag{2}$$

Two regimes of electron beam interaction with the beat wave can be distinguished, as indicated in Fig. 2. The sketches in the top row correspond to

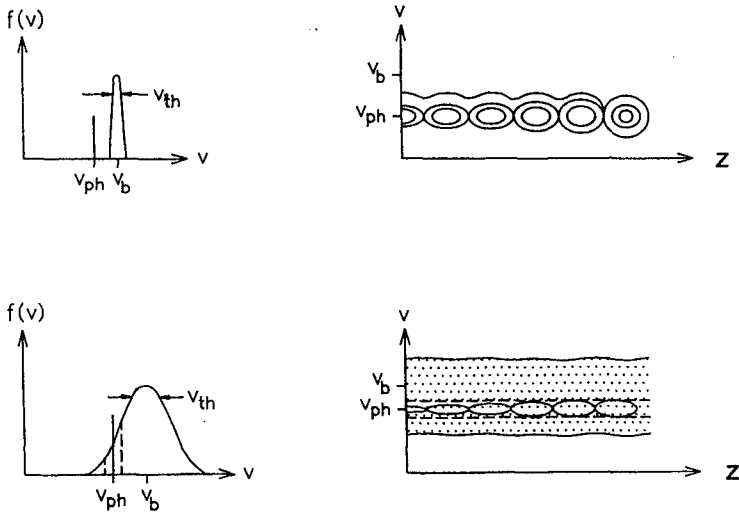


FIGURE 2. Velocity distribution function & longitudinal phase space. Upper panel: Cold beam regime. Lower panel: Warm beam regime.

interaction in the cold beam regime wherein the phase velocity lies outside of the electron distribution function. In phase space the electrons interact with and lose energy as the ponderomotive buckets grow with propagation distance, eventually becoming trapped in the buckets. In contrast, in the warm beam regime, the lower pair of plots in Fig. 2, only a fraction of the electrons participate in the interaction. Clearly, for high efficiency operation the cold beam regime is preferable as it permits a larger extraction from the electron beam.

Kinetic analysis of the FEL is based on the Vlasov-Maxwell system of equations. The equilibrium is constructed from the constants of motion and the perturbed distribution function is obtained by integration over unperturbed orbits [4]. Fundamentally, the electron equilibrium orbits consist of two oscillatory contributions. The fine scale motion is the wiggle motion, while betatron oscillations take place over a much longer scale, as shown in Fig. 3. The wiggle motion is due to the alternating polarity of the magnets comprising the wiggler and can be averaged over. Betatron oscillations arise since in a realizable wiggler the magnetic field is larger near the pole faces. An electron that approaches the pole face experiences an increasing field that deflects it back towards the axis. The initial transverse phase space of the electron beam (i.e., emittance) determines the range of betatron orbits in the wiggler. Writing the axial electron velocity (normalized to c) as

$$\beta_z = \beta_{z0} + \delta\beta_z, \quad (3a)$$

where

$$\beta_{z0} = 1 - \frac{1 + a_w^2/2}{2\gamma_0^2} \quad (3b)$$

is common to all electrons, the deviation in the axial velocity is expressible as

$$\delta\beta_z = \frac{1 + a_w^2/2}{\gamma_0^2} \frac{\delta\gamma}{\gamma_0} - \frac{1}{2} k_{\beta 0}^2 y_\beta^2 + \frac{k_{p0}^2}{2\beta_{z0}\gamma_{z0}\gamma_0^{1/2}} y_\beta^2. \quad (3c)$$

Here, $k_{p0} = (4\pi n_0 |e|^2 / mc^2)^{1/2}$ and $k_{\beta 0} = a_w k_w / 2^{1/2} \gamma_0 \beta_{z0}$ are the plasma and betatron wavenumbers, respectively, n_0 is the density, y_β is the betatron oscillation amplitude and $a_w = |e|A_w / mc^2$ is the normalized wiggler amplitude, with A_w defined below. [4]. The first term in Eq. (3c) is due to intrinsic energy spread on the beam from the accelerator. The second term is due to betatron oscillations and the last term arises from space charge effects.

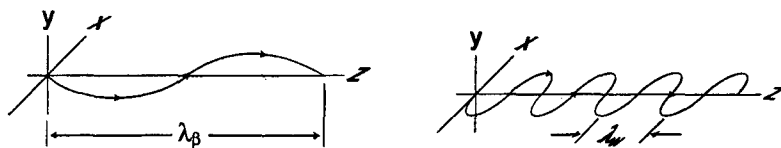


FIGURE 3. Left: Betatron oscillations in vertical plane. Right: Wiggle motion in horizontal plane.

A physical process that affects FEL operation significantly is related to diffraction of the radiation beam. A 2- or 3-D theory is necessary to take account of diffraction. Typically a laser beam is brighter in the center than at the edges and tends to spread in the transverse direction. Electron beams also have a similar profile and tend to spread laterally due to emittance. The combined effect of these is that the growth rate of the instability is higher in the central portion. This leads to the phenomenon of gain guiding—leading to the impression that the laser beam is guided, whereas in fact the diffractive loss is constantly compensated by higher growth near the center of the beam.

The wiggler vector potential is given by,

$$\mathbf{A}_w = A_w \cosh(k_w y) \sin(k_w z) \mathbf{e}_x$$

while the vector potential of the radiation field has the form

$$\mathbf{A}_s = \frac{1}{2} A_s(y, z) \exp[i(kz - \omega t)] \mathbf{e}_x + c.c.$$

The equilibrium electron beam distribution function is constructed from the constants of motion and has the form,

$$F(E, P_x, J) = \frac{I_b / (\beta_{z0} I_0)}{2\pi m c r_e \varepsilon_N} \frac{\exp[-(E - E_0)^2 / (\sigma_y m c^2)^2]}{\sqrt{\pi} \sigma_y m c^2} \\ \times \delta(P_x) \exp(-k_{\beta 0} J / m c \varepsilon_N k_{\beta})$$

where $J = \iint dy dp_y / 2\pi$ is the area of transverse phase space, ε_N is the normalized emittance and σ_y is the energy spread.

EXAMPLES

To illustrate the effects of beam quality on FEL operation and, in particular, the utility of the scaled thermal velocity in identifying the regime of operation, three examples will be discussed in the following.

In the first example, the scaled growth rate $\text{Im } \mu/D$ and S are plotted as a function of scaled emittance $k_s \epsilon$ in Fig. 4. Observe that the growth rate decreases with emittance, as expected. Over the same range of emittance, S starts out from very small values and passes through unity for $k_s \epsilon \approx 1/3$ and then increases rapidly thereafter indicating transition into the warm beam regime.

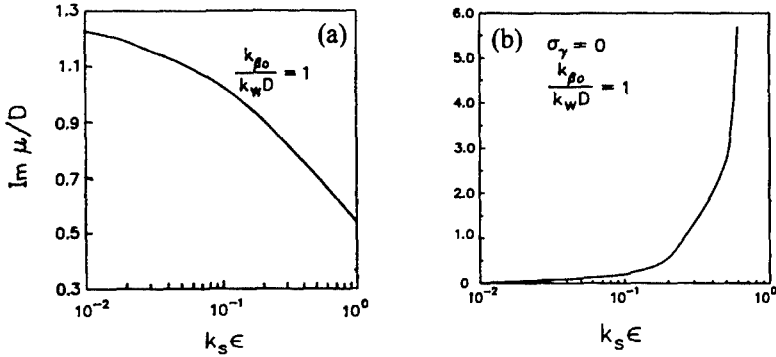


FIGURE 4. (a): Scaled growth rate versus scaled emittance. (b): Scaled thermal velocity versus scaled emittance.

The next example is useful since it can be related to a problem of current interest. Namely, the growth rate of FELs in the X-ray regime can be very small if GeV class electron beams are utilized. Physically this is due to the fact that at high energies electrons are very stiff with very small quiver motion. To compensate for this the electron beam current can be increased to enhance the growth rate. There are several techniques for increasing electron beam current, using RF beams, chicanes, etc. to rotate electron bunches in longitudinal phase space, as indicated in Fig. 5.

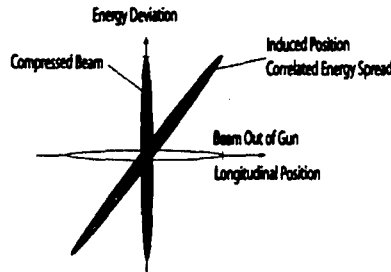


FIGURE 5. Bunch rotation & compression in longitudinal phase space.

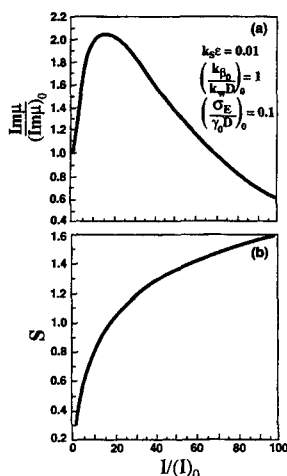


FIGURE 6. (a) Growth rate ratio & (b) scaled thermal velocity versus compression ratio

By rotating the electron bunch the length of the bunch is reduced and thus the current goes up. But by Liouville's theorem the axial momentum spread has to increase. Since the latter tends to spoil the FEL resonance it is not *a priori* obvious if longitudinal beam compression is useful. To study this Fig. 6 shows a plot of the growth rate of the compressed beam $\text{Im} \mu$ normalized to that for an uncompressed beam $(\text{Im} \mu)_0$ as a function of the compression ratio. The compression ratio is the same as the ratio of current in the compressed beam to that in the uncompressed beam $I/(I)_0$. Figure 6 shows that for relatively small compression the growth rate is in fact increased. Eventually, however, the increased spread in axial momentum dominates and the growth rate no longer increases. Not surprisingly, the optimal compression is seen to correspond to $S \approx 1$.

As a final example it is interesting to consider a compact (e.g. bench top) FEL. The lasing wavelength in an FEL is given by

$$\lambda = \frac{1 + a_w^2/2}{2\gamma_0^2} \lambda_w. \quad (4)$$

Planned X-ray free electron laser experiments at SLAC and DESY employ wigglers with cm-scale periods and GeV electron beams to generate radiation at X-ray wavelengths. As can be seen from Eq. (4) one may also obtain lasing in the X-ray regime by using μm -scale wiggler periods along with modest energy (10's of MeV) electron beams. Such fine-scale wigglers are readily available in the form of T^3 lasers. The electromagnetic field of a laser beam can induce wobble motion of electrons just as well as the magnetic field of a conventional wiggler. The only difference between a magnetostatic wiggler and an electromagnetic wiggler is that in the latter case the factor-of-2 in the denominator of Eq. (4) is to be replaced by 4. At

low voltages, however, space charge can become an issue. Fortunately there appears to be a solution to this that at first sight is rather surprising. It turns out that there can be an equilibrium flow of a finite-emittance electron beam in a wiggler wherein space charge forces, wiggler focusing and emittance are balanced. For this matched beam the spread in axial velocity due to wiggler gradients and space charge [the last two terms in Eq. (3c)] tend to cancel.



FIGURE 7. Electron beam transverse profile versus propagation distance. Electron gun is to the left. Upper panel: Betatron oscillation in absence of space charge. Lower panel: Nearly matched electron beam with self-field parameter = 0.85.

Figure 7 shows an example where space charge effects are included (lower panel) and excluded (upper panel). The self-field parameter is defined by $SFP = [(k_p / k_{\beta 0}) / (\gamma_{z0} \gamma_0^{1/2} \beta_{z0})]^2$. With no space charge the beam undergoes betatron oscillations. With an appropriate amount of charge in the bunch the beam is nearly matched and $\delta\beta_z$ is reduced [5], beam quality is improved and transition to the warm beam regime takes place for much larger emittance, as indicated in Fig. 8.

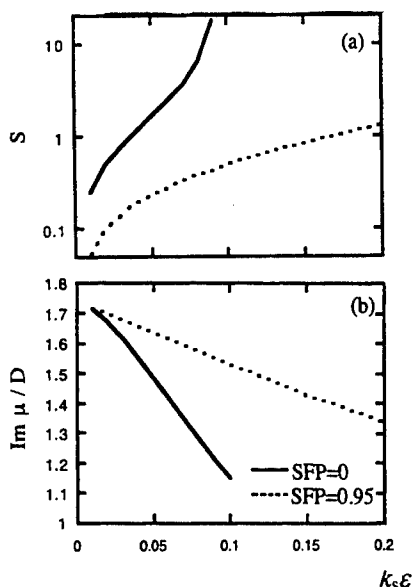


FIGURE 8. (a) Scaled thermal velocity & (b) growth rate versus scaled emittance. Dashed curves correspond to no space charge. Solid curves include space charge effects with $SFP = 0.95$.

In summary it is shown that the scaled thermal used in Ref. 3, when written in the more general form $S \equiv v_{th} / (v_b - v_{ph})$, makes an excellent definition of electron beam quality for FELs. For example, using this definition it is possible to determine if beam compression will increase or decrease the growth rate. When S is plotted as a function of emittance one can readily ascertain if the FEL is in the kinetic regime, with substantially reduced growth rate. Finally, it is shown that there is an unusual mode of operation when space charge effects are important and yet beam quality is improved.

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